

# Effective description of brane terms in extra dimensions

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**ABSTRACT:** We study how theories defined in (extra-dimensional) spaces with localized defects can be described perturbatively by effective field theories in which the width of the defects vanishes. These effective theories must incorporate a “classical” renormalization, and we propose a renormalization prescription *à la* dimensional regularization for codimension 1, which can be easily used in phenomenological applications. As a check of the validity of this setting, we compare some general predictions of the renormalized effective theory with those obtained in a particular ultraviolet completion based on deconstruction.

**KEYWORDS:** Beyond Standard Model, Large Extra Dimensions, Field Theories in Higher Dimensions.

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## 1. Introduction

Many physical systems are described by fields propagating in a space with lower-dimensional defects, including, in particular, boundaries. These infinitely thin defects are typically idealizations of localized physical backgrounds with finite size and a certain substructure. The field theory should then be regarded as an effective theory valid at low energies, such that the substructure of the defects is not resolved. An implicit assumption underlying the simplification of using zero-width (“thin”) defects is that at low energies all observables are fairly insensitive to ultraviolet details. As we will see, this is not always the case: there are examples in which the details of the defects do not decouple but filter into the low-energy observables. Nevertheless, we will argue that in all cases an effective field theory with thin defects can describe low-energy physics to any required precision. The only difference between the decoupling and the non-decoupling scenarios is that in the second case (part of) the substructure of the brane is described by relevant operators.

On the other hand, it turns out that perturbative calculations in the presence of thin defects are often plagued with extra divergences that arise in the limit of zero thickness. In some cases they appear already at the classical level. These divergences signal a breakdown

of the field theory at scales where the finite thickness of the defects cannot be neglected. They must be renormalized away, and the information about the microscopic structure of the defects is then encoded in the renormalized coefficients of the different operators of the theory.

In this paper we propose a simple renormalization prescription to deal with the divergences associated to thin defects (thin-brane divergences), and study the effect of bulk and localized higher-order terms. It turns out that, in our scheme, the most singular (orthogonal) localized kinetic terms can be completely eliminated via field redefinitions. This justifies the conventional phenomenological approach of ignoring them. Furthermore, we check that the results obtained in the effective framework agree with those given by a particular (deconstructed) microscopic theory. The latter will be described in detail in a forthcoming publication [1].

The paper is organized as follows. In section 2, we argue that effective theories with infinitely thin defects are a good description of more fundamental theories, in which the defects may have some internal structure. We also introduce the particular setup to be studied in the following sections. In section 3, we describe our renormalization scheme and show how it can be used to eliminate some operators to all orders. In section 4 we calculate, to second order in the derivative expansion, the spectrum and wave functions of fermions, scalars and gauge bosons for a general effective theory. Section 5 contains the matching to the deconstructed models. Section 6 is devoted to a particular class of operators, which are ambiguous in the limit of zero brane width. We present our conclusions in section 7. Finally, in appendix A we give an explicit regularization which realizes our renormalization prescription and in appendix B we give details of an extra test of our prescription, five-dimensional Green-Schwarz mechanism in the vector formulation.

## 2. Effective theories with thin branes

In the absence of defects, and under very general assumptions, any fundamental theory can be described, at energies below some scale  $\Lambda_0$ , by an effective quantum field theory for the light degrees of freedom [2, 3]. This  $\Lambda_0$  is related to some characteristic dimensionful parameter of the fundamental theory; it can also represent a scale at which the effective theory becomes strongly coupled. The subscript 0 is used to distinguish this scale from the cutoff of the effective theory with defects. The effective Lagrangian can be expanded in an infinite series of local operators, organized in powers of  $E/\Lambda_0$  and  $m_i/\Lambda_0$ , where  $E$  is the energy and  $m_i$  represent possible mass scales in the theory, smaller than  $\Lambda_0$ . Of course, at energies below a given  $m_i$  one could describe physics by a new effective theory with  $\Lambda'_0 = m_i$ . In a Wilsonian framework, the scale  $\Lambda_0$  represents some cutoff of external and virtual momenta, such that the effective theory cannot resolve distances smaller than  $\Lambda_0^{-1}$ . In practice, however, it is more convenient to work with effective theories renormalized in a mass-independent scheme, such as dimensional regularization with minimal subtraction. The main reason is that this prevents divergent loop corrections from enhancing the effect of higher-order operators, so that operators of order greater than a given  $n$  do not contribute to observables to order  $\Lambda_0^{-n}$ . Another reason is that preserving symmetries in a Wilsonian

context is more involved. In a mass-independent scheme, the scale  $\Lambda_0$  only appears in the effective theory in the explicit inverse powers in front of the different operators.

Consider now a generic “fundamental” theory defined in some flat space that contains defects extended in  $D$  infinite space-time dimensions, with a characteristic thickness  $\epsilon_0$ , much smaller than the size of the transverse dimensions  $L$ . Furthermore, we restrict to plane defects with vanishing extrinsic curvature. These requirements are not essential but simplify the discussion. The defects can have different microscopic origins: surfaces of materials, solitonic configurations, orientifold planes, D-branes, intersections of D-branes, etc. In the following we will generically call these objects “branes”. For simplicity, we assume that all other scales characterizing the brane (such as possible dimensionful couplings between a localized background and the fluctuating fields) are of the order of  $\epsilon_0^{-1}$  as well. Because Poincaré invariance in the transverse directions is broken, the corresponding momenta are no longer good quantum numbers. It is then convenient to work in position space for the transverse coordinates, and to speak of derivatives, rather than energies. D-momenta and Kaluza-Klein (KK) masses can also label the eigenstates of the free Hamiltonian, but they are not adequate to organize local operators. We assume that, without branes, an effective description exists in which the cutoff  $\Lambda_0$  is larger than the compactification scale  $L^{-1}$ . The latter can then be thought of as one of the low scales  $m_i$ .

When the branes are introduced, we can distinguish two physical situations, according to the relative sizes of  $\Lambda_0$  and  $\epsilon_0^{-1}$ . If  $\epsilon_0^{-1} < \Lambda_0$ , it is possible in principle to describe physics at energies below  $\Lambda_0$  by an effective field theory incorporating a field-theoretical representation of the branes at scales between  $\epsilon_0^{-1}$  and  $\Lambda_0$ . One example of this situation is the calculation of zero-point (Casimir) energies of quantum electromagnetic fields in a conducting cavity or in the presence of conducting plates, and toy models related to this situation.<sup>1</sup> If, instead,  $\epsilon_0^{-1} \gtrsim \Lambda_0$  then the microscopic structure of the branes lies beyond the reach of the effective field theory, and can only be described at the level of the fundamental theory. This situation is implicit in many field-theoretical models in extra dimensions. In the example of fundamental theory we will consider below, deconstruction, the scale  $\Lambda_0$  (which is identified with the inverse of the lattice spacing) acts as a hard cutoff in position space, which smears the brane over an effective size  $\Lambda_0^{-1}$ .

In all cases the theory can be described at energies lower than  $\Lambda = \min\{\Lambda_0, \epsilon_0^{-1}\}$  by an effective field theory with cutoff  $\Lambda$ .<sup>2</sup> Note that if we send the physical scale  $\epsilon_0^{-1}$  to infinity

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<sup>1</sup>See [4] for a discussion of effective theories and matching in the context of the Casimir effect, and [5] for calculations with “fat” branes. Note that the localized energy density is a relevant operator in the effective theory below  $\Lambda = \epsilon_0^{-1}$  (with thin branes represented by boundary conditions). Its coefficient, which is naturally controlled by the scale  $\epsilon_0^{-1}$ , is an input parameter to be fixed by experiment or by matching with the theory with finite  $\epsilon_0$ .

<sup>2</sup>When  $\epsilon_0^{-1} < \Lambda_0$  we could use separate cutoffs for brane and bulk operators:  $\Lambda_{\text{brane}} = \epsilon_0^{-1}$ ,  $\Lambda_{\text{bulk}} = \Lambda_0$ . This is analogous to the position-dependent cutoff that is used in warped geometries. Locality of the ultraviolet divergences implies that coefficients of brane operators do not appear in the running of the coefficients of bulk operators. The converse does not necessarily hold, but the suppressions by powers of  $1/\Lambda_{\text{brane}}$  are not destabilized by the addition of powers of  $1/\Lambda_{\text{bulk}}$ . Nevertheless we stick to the effective theory with a single cutoff  $\Lambda$ , although additional suppressions by powers of  $1/(\epsilon_0\Lambda_0)$  could be expected for dimensionless bulk couplings.

the branes do not disappear, but become infinitely thin. Correspondingly, the effective theory lives in a space with “effective” branes of size  $\epsilon \leq \Lambda^{-1} < L$ . We stress that the auxiliary thickness  $\epsilon$  of the branes in the effective theory is not necessarily related to the physical thickness  $\epsilon_0$  of the branes in the fundamental theory (which in some scenarios is represented by  $\Lambda$ ). Actually, at the end of the day we will send  $\epsilon \rightarrow 0$  to describe theories with finite  $\epsilon_0$ .

One important feature of theories with branes is the appearance of localized divergent radiative corrections, which implies that brane localized terms must be included in the theory for multiplicative renormalizability [6–8]. In other words, putting them to zero is not stable under renormalization group evolution. These brane operators can also be present at the scale  $\Lambda$ , for instance if they are radiatively generated by heavy degrees of freedom which have been integrated out. Operators with the same field content are organized according to their canonical dimension (i.e. the number of derivatives and delta functions). This corresponds to an expansion in  $1/\Lambda$ . In the Wilsonian framework, because the cutoff makes the theory insensitive to distances smaller than  $\Lambda^{-1}$ , we can use effective branes of any shape and size  $\epsilon$  we like, as long as the integrated features of the original branes over a region of size  $\Lambda^{-1}$  are preserved. If we knew the fundamental theory, the process of integrating out the degrees of freedom higher than  $\Lambda$  would naturally give  $\epsilon \approx \Lambda^{-1}$ . On the other hand, in a mass-independent scheme the theory will be sensitive to the auxiliary scale  $\epsilon^{-1}$ . In this framework we would like to keep  $\Lambda$  as the only dimensionful scale (outside loop logarithms), and eliminate the auxiliary brane thickness by taking the thin-brane limit  $\epsilon \rightarrow 0$ . However, this is not straightforward, for this limit can be divergent (after subtraction of the usual divergences of correlators at coincident points). The divergences arise because in the presence of certain brane terms, the fields (even for the lowest KK modes) fluctuate very strongly near the branes, in such a way that the local value of the derivatives of the fields is of order  $\epsilon^{-1}$ . Hence, perturbativity in  $\partial/\Lambda$  is spoiled. The solution is, as usual in field theory, to apply a renormalization procedure to eliminate the dependence on  $\epsilon$ . The limit  $\epsilon \rightarrow 0$  can then be safely taken. Furthermore, to keep the virtues of the “quantum” mass-independent scheme, we should use a mass-independent scheme also for the thin-brane divergences.

This kind of effective theory can in principle describe any sensible fundamental theory defined in manifolds with branes. These can be classified into universality classes, with theories within the same class being described by the same effective theory at lowest order. Sometimes a small perturbation in the ultraviolet of the fundamental theory can turn out to be relevant and bring the theory to a different universality class. We will show examples of this situation below.

In this paper we are mainly interested in field theories in more than four dimensions with branes. Extra-dimensional quantum field theories are non-renormalizable, so they are necessarily effective theories, even in the absence of branes. Because of the power-law running of the couplings, the cutoff  $\Lambda_0$  cannot be much larger than the compactification scale  $L^{-1}$  if we are to stay in a perturbative regime and, at the same time, reproduce the observed gauge couplings. On the other hand, the substructure of the branes is often assumed to be described by some fundamental theory such as string theory. In this case,

they belong to the class with  $\epsilon_0^{-1} \gtrsim \Lambda_0 = \Lambda$ . In extra dimensions, non-renormalizability implies that an infinite number of localized operators are generated. At leading order in the low-energy expansion they consist of localized mass terms, kinetic terms and marginal interactions [8, 9]. Some of these brane terms, in turn, give rise to singularities and a loss of perturbativity in the thin-brane limit, unless they are subtracted. Such a renormalization has been proposed and studied in [10] for branes of codimension 2 with localized mass terms, and in [11, 12] for branes of codimension 1 with localized derivative (kinetic) terms. This renormalization is usually dubbed “classical” because it is required already at tree level in the presence of tree-level brane terms.

Our purpose here is to study the effect of brane and bulk operators to second order in perturbation theory (up to  $\Lambda^{-2}$ ), and to give a simple renormalization prescription, which can be used in phenomenological calculations with thin branes. Furthermore, we want to check that the results obtained with this prescription are physical, in the sense that they agree with the ones given by a more fundamental theory incorporating a microscopic description of the branes. We consider theories with plane parallel branes of codimension 1 and, for definiteness, restrict to the orbifold  $M_4 \times S^1/Z_2$ , which we parametrize (in the “upstairs” picture) by  $x^\mu$ ,  $\mu = 0, 1, \dots, 3$  and  $y = x^5 \in (-\pi R, \pi R]$ . Our branes are the fixed points of the  $Z_2$  action, located at  $y = 0$  and  $y = \pi R$ . Therefore, they are non-dynamical objects and we do not need to include their fluctuations (“branons”, see [13–15]) in the effective theory. We study the free theory, which is already non-trivial, and concentrate on the kinetic terms, which are the most relevant in phenomenology. We give a general basis of independent operators which, in principle, can describe to a certain order any ultraviolet completion with the assumed symmetries in this sector. Then, we study the impact of these operators on the KK spectrum in a perturbative calculation. Finally, we match the general (free) effective theory to a specific completion: deconstructed orbifolds [1]. The fact that this matching is possible is a check of the validity of the effective framework, of classical renormalization, and of our particular prescription. The description of the free part of brane fields is straightforward in flat space for plane branes, so we will focus mostly on bulk fields.<sup>3</sup> We choose to work in the parent theory with fields that have well-defined orbifold parity, rather than in the interval with boundary conditions.

### 3. Renormalization

We want an effective theory with branes of vanishing width, i.e. , with the brane-localized terms proportional to Dirac delta functions. We will work formally with these representations whenever possible, and only resort to an intermediate regularization of the delta functions to check the results in section 6. Since products of delta functions appear in perturbation theory, even classically, a subtraction procedure is in order. We propose a simple prescription to perform the subtractions: define all products of delta functions or their

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<sup>3</sup>Brane fields contribute to brane-localized free terms for bulk fields via quantum corrections. At the classical level, they can contribute as well if there is mass or kinetic mixing with the bulk fields. Actually, we will consider below one case in which a brane-bulk fermionic mass mixing has dramatic effects. Elsewhere, diagonal mass and kinetic matrices are assumed.

derivatives as identically zero. This applies to products of deltas both in the action and in the calculations of amplitudes (or KK reduction), and determines a mass-independent renormalization scheme, which we call **analytic renormalization**. The reason for the term “analytic” is that, in particular, this prescription follows automatically from an analytic regularization involving (extensions of) zeta-functions, which we describe in appendix A. In practice, however, no explicit realization is needed in our calculations for plane branes of codimension 1, where the thin-brane divergences are power-like. Nevertheless, as we discuss below, an explicit regularization could be useful to deal with finite ambiguous contributions, which appear also in the plane case. Moreover, logarithmic divergences appear in codimension 2 [10, 16, 17], and when the branes are curved, extra localized terms proportional to their extrinsic curvature arise after careful regularization of the singularities [18]. Therefore, a refinement of analytic renormalization (may be some form of differential renormalization [19]), or an explicit (analytic) regularization, is required in these and other more general situations. Our prescription (and the resulting renormalization scheme) presents several advantages:

- It is extremely simple, as no intermediate regularization of the Dirac deltas is necessary and no explicit counterterms have to be computed.
- It does not introduce any dimensionful regulator which could interfere with the expansion in inverse powers of  $\Lambda$ .
- As we show below, it allows the elimination of some brane terms via field redefinitions.
- It preserves supersymmetry, at least in known examples.

Observe that even if the thin-brane divergences signal a dependence of ultraviolet details, we can cancel them completely since the relevant information is encoded in the finite coefficients of operators with at most a single delta function. Putting all products of (derivatives of) deltas to zero is equivalent to exactly cancelling these products by counterterms in the renormalized action, as proposed in [12], but it is simpler, as many terms in amplitudes or in the action can be discarded from the beginning. For instance, and in relation to the last point, it is known that in supersymmetric theories with branes higher-order terms with products of delta functions have to be included in the action to preserve supersymmetry [20, 12]. But if analytic renormalization is used in all calculations, these terms can (and should) be omitted. This is possible because products of deltas are also renormalized to zero in the on-shell supersymmetry transformations. In particular, the free action for a supersymmetric boson with brane terms, which includes an infinite series of higher-order terms [12], is equal to the naïve one after renormalization. This means that the latter, when combined with the free fermionic action, is supersymmetric with our prescription. We shall exploit this fact below.

Although discarding divergent terms is just a device to save work in the cases in which they eventually cancel [20, 21], we note that, as with other regularizations, one must be careful in identifying the divergent  $\delta(0)$ 's, so as not to leave a finite remnant which could lead to inconsistencies or to broken symmetries. In this regard, and as another

example of our renormalization scheme, we have shown in appendix B that our prescription reproduces the correct results in the vector field formulation of the five-dimensional Green-Schwarz mechanism, which was studied in [22].<sup>4</sup> On the other hand, throwing away these divergences does have physical content in truly divergent situations, where it amounts to a renormalization.

Analytic renormalization allows us to eliminate many operators in the effective action using field redefinitions. Consider for instance a kinetic Lagrangian for fermions with general (lowest-order) brane terms at one of the fixed points:

$$\begin{aligned} \mathcal{L} = & (1 + a^L \delta_0) \bar{\chi}_L i \not{\partial} \chi_L + (1 + a^R \delta_0) \bar{\chi}_R i \not{\partial} \chi_R - \frac{1}{2} (1 + b^L \delta_0) (\bar{\chi}_L \partial_5 \chi_R + (\partial_5 \bar{\chi}_R) \chi_L) \\ & + \frac{1}{2} (1 + b^R \delta_0) (\bar{\chi}_R \partial_5 \chi_L + (\partial_5 \bar{\chi}_L) \chi_R), \end{aligned} \quad (3.1)$$

with  $\delta_0 = \delta(x^5)$  and  $\chi_{L,R}$  the left-handed and right-handed chiral projections of five-dimensional (four-component) Dirac spinors. The brane kinetic terms with derivatives normal to the branes (“orthogonal” brane terms), with coefficients  $b^{L,R}$ , give rise to thin-brane singularities in the classical propagator [12]. Performing a field redefinition  $\chi_c = h_c \psi_c$  with  $h_c = (1 + \frac{b^L + b^R}{2} \delta_0)^{-\frac{b^c}{b^L + b^R}}$  and  $c = L, R$ , the free Lagrangian is written as

$$\mathcal{L} = (1 + a^L \delta_0) h_L^2 \bar{\psi}_L i \not{\partial} \psi_L + (1 + a^R \delta_0) h_R^2 \bar{\psi}_R i \not{\partial} \psi_R - \bar{\psi}_L \partial_5 \psi_R + \bar{\psi}_R \partial_5 \psi_L. \quad (3.2)$$

We have traded the orthogonal brane kinetic terms for parallel brane kinetic terms (those without normal derivatives) times singular expressions. This makes the divergences associated to orthogonal brane terms apparent. But with analytic renormalization, (3.2) reduces to

$$\mathcal{L} = (1 + \bar{a}^L \delta_0) \bar{\psi}_L i \not{\partial} \psi_L + (1 + \bar{a}^R \delta_0) \bar{\psi}_R i \not{\partial} \psi_R - \bar{\psi}_L \partial_5 \psi_R + \bar{\psi}_R \partial_5 \psi_L, \quad (3.3)$$

with  $\bar{a}^c = a^c - b^c$ , which contains only non-singular parallel brane terms. This is equivalent to performing the first-order field redefinitions of ref. [12] and discarding all the higher-order terms that are generated.

#### 4. KK decomposition of renormalized effective theories

Next we write down general free effective Lagrangians for massless fermions, scalars and gauge bosons to order  $\Lambda^{-2}$  and perform the KK reduction to the same order. We impose 4D Lorentz invariance in the directions parallel to the branes. We also impose the full 5D Lorentz invariance in the bulk at zeroth order<sup>5</sup> but allow for its breaking in higher-order bulk operators. This is necessary to describe ultraviolet completions breaking this symmetry, such as deconstruction, and can be useful in model building [23]. In the following we will refer to the brane terms, i.e. to operators with a delta function, as odd-odd when they involve products of odd functions and as even-even otherwise. The operators are

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<sup>4</sup>We thank the referee for bringing our attention to this reference and suggesting this test.

<sup>5</sup>Note that this is automatic in the free theory if there is only one particle, as can be seen by a redefinition of the coordinate  $y$ .



always invariant, so the number of odd factors must be even. Note that odd-odd operators are ambiguous. As we shall see in the next subsections, they do not contribute to second order. We discuss their impact on higher-order corrections in section 6.

### 4.1 Fermions

In the fermionic case we also allow for operators proportional to the background  $\sigma(y) = \text{sign}(y)$ , because they can mimic the effect of a Wilson term in deconstruction, and for a breaking of chiral invariance (without masses). The free fermion Lagrangian can be written as  $\mathcal{L}_f = \mathcal{L}_f^{(0)} + \frac{1}{\Lambda}\mathcal{L}_f^{(1)} + \frac{1}{\Lambda^2}\mathcal{L}_f^{(2)} + \dots$ , where

$$\mathcal{L}_f^{(0)} = \bar{\psi}(i\partial - \gamma_5\partial_5)\psi, \tag{4.1}$$

$$\mathcal{L}_f^{(1)} = \kappa_1\sigma(\partial_5\bar{\psi})\partial_5\psi + a_I^R\delta_I\bar{\psi}_R i\partial\psi_R + a_I^L\delta_I\bar{\psi}_L i\partial\psi_L, \tag{4.2}$$

$$\mathcal{L}_f^{(2)} = \kappa_2\bar{\psi}_L\partial_5^3\psi_R + \xi_I\sigma\delta_I(\partial_5\bar{\psi}_L)\partial_5\psi_R + \eta_I^L\sigma\delta_I\bar{\psi}_L\partial_5^2\psi_R + \eta_I^R\sigma\delta_I\bar{\psi}_R\partial_5^2\psi_L + \text{h.c.} \tag{4.3}$$

Here,  $\partial = \partial_\mu\gamma^\mu$ ,  $\delta_I = \delta(y - RI)$ , with  $I = 0, \pi$  labelling the positions of the fixed points, and sums over repeated indices  $I$  are understood. We choose  $\psi_R$  ( $\psi_L$ ) to be even (odd) under the orbifold parity. All the parameters, except  $\Lambda$ , are dimensionless. Several possible operators have been eliminated by integration by parts, use of the zeroth-order equations of motion (or equivalently, perturbative field redefinitions) and analytic renormalization. The values  $\kappa_1 = \kappa_2 = 0$  correspond to 5D Lorentz invariance in the bulk. We have chosen a basis of operators which leads to a convenient KK reduction, such that the resulting 4D theory has no higher-derivatives in the kinetic term. Indeed, if we expand  $\psi_{L,R}(x, y) = \sum_n f_n^{L,R}(y)\Psi_{L,Rn}(x)$  and take  $f_n^{L,R}$ ,  $m_n$  to be the eigenvectors and eigenvalues of the generalized eigenvalue problem

$$\begin{aligned} [-\partial_5 + \frac{\kappa_1}{\Lambda}\partial_5\sigma\partial_5 + \frac{\kappa_2}{\Lambda^2}\partial_5^3]f_n^L &= m_n(1 + \frac{a_I^R}{\Lambda}\delta_I)f_n^R, \\ [\partial_5 + \frac{\kappa_1}{\Lambda}\partial_5\sigma\partial_5 - \frac{\kappa_2}{\Lambda^2}\partial_5^3]f_n^R &= m_n f_n^L, \end{aligned} \tag{4.4}$$

with normalization

$$1 = \int_{-\pi R}^{\pi R} dy \left(1 + \frac{a_I^R}{\Lambda}\delta_I\right) (f_n^R)^2 = \int_{-\pi R}^{\pi R} dy (f_n^L)^2, \tag{4.5}$$

the free Lagrangian reduces to

$$\mathcal{L}_f = \sum_n \bar{\Psi}_n(i\partial - m_n)\Psi_n, \tag{4.6}$$

with  $\Psi_n = \Psi_{Ln} + \Psi_{Rn}$ , and  $\Psi_{L\bar{n}} = 0$  ( $\Psi_{R\bar{n}} = 0$ ) for possible right-handed (left-handed) modes with  $m_{\bar{n}} = 0$ . In writing the eigensystem (4.4) we have used the fact that the terms in the action with coefficients  $a_I^L$ ,  $\xi_I$ ,  $\eta_I^L$  and  $\eta_I^R$  do not contribute to second order, as they vanish when the (continuous) zeroth-order wave functions are used. Then, to second order we can safely work with strict delta functions and the calculation is straightforward: at each order we solve the bulk equation and apply the boundary (“jump”) conditions found by integrating around the fixed points.

Expanding the KK masses and wave functions in  $1/\Lambda$  we find a flat right-handed zero mode plus a tower with KK masses

$$m_n = \frac{n}{R} \left[ 1 + A \frac{1}{R\Lambda} + (A^2 + Bn^2) \frac{1}{(R\Lambda)^2} \right] + \dots, \quad (4.7)$$

where  $A = -\frac{a_0^R + a_\pi^R}{2\pi}$ ,  $B = \frac{\kappa_2^2}{2} + \kappa_2$  and  $n = 1, 2, \dots$ . The structure of (4.7), with a piece proportional to  $n$  depending on a single number  $A$  and another piece proportional to  $n^3$ , which appears at second order, is a consequence of the symmetries we have imposed on the effective Lagrangian, of the fact that the expansion in local operators is controlled by a single scale  $\Lambda$ , and of analytic renormalization, which does not mix up this ordering. The wave functions of massive modes in the fundamental region have the structure

$$f^c(y) = P_1^c(y) \cos\left(\frac{n}{R}y\right) + P_2^c(y) \sin\left(\frac{n}{R}y\right), \quad (4.8)$$

with  $P_{1,2}^c$  polynomials of second degree (to second order in  $\Lambda^{-1}$ ). Here we write them to first order only:

$$\begin{aligned} P_1^R(y) &= N_0 + N_1 \frac{1}{R\Lambda}, \\ P_2^R(y) &= -N_0 \frac{n}{R\Lambda} \left( \frac{a_0^R}{2} + A \frac{y}{R} \right), \\ P_1^L(y) &= -N_0 \frac{n}{R\Lambda} \left( \frac{a_0^R}{2} + \kappa_1 + A \frac{y}{R} \right), \\ P_2^L(y) &= -N_0 - N_1 \frac{1}{R\Lambda}, \end{aligned} \quad (4.9)$$

with  $N_0$  and  $N_1$  perturbative normalization constants. Observe that the parameter  $\kappa_1$  only appears in the KK masses in the combination  $\frac{\kappa_1^2}{2} + \kappa_2$  to second order. On the other hand, the first-order wave function for the left-handed component depends on  $\kappa_1$  but not on  $\kappa_2$ ; therefore, the operator with coefficient  $\kappa_1$  is not redundant. The wave functions also distinguish  $a_0^R$  from  $a_\pi^R$  at first order.

## 4.2 Scalars

For a massless complex scalar, after integration by parts, field redefinitions and analytic renormalization, the effective Lagrangian to second order reads  $\mathcal{L}_s = \mathcal{L}_s^{(0)} + \frac{1}{\Lambda} \mathcal{L}_s^{(1)} + \frac{1}{\Lambda^2} \mathcal{L}_s^{(2)} + \dots$ , with

$$\mathcal{L}_s^{(0)} = \phi^\dagger (-\square + \partial_5^2) \phi, \quad (4.10)$$

$$\mathcal{L}_s^{(1)} = a_I \delta_I \phi^\dagger \square \phi - c_I \delta_I (\partial_5 \phi^\dagger) \partial_5 \phi, \quad (4.11)$$

$$\mathcal{L}_s^{(2)} = \kappa \phi^\dagger \partial_5^4 \phi, \quad (4.12)$$

and  $\square = \partial_\mu \partial^\mu$ . We have not included terms proportional to  $\sigma$  in the scalar case, as they are not required to reproduce the results in deconstruction. A possible orthogonal brane kinetic term  $b_I \delta_I \phi^\dagger \partial_5^2 \phi + \text{h.c.}$  has been absorbed into the  $a$ -term, using field redefinitions

and analytic renormalization. If 5D Lorentz invariance is preserved in the bulk, then  $\kappa = 0$ . The KK reduction is performed by expanding  $\phi(x, y) = \sum_n f_n(y)\Phi_n(x)$  with  $f_n$  the eigenfunctions of the eigenvalue problem

$$\mathcal{O}_s f_n = -m_n^2(1 + a_I \delta_I) f_n, \quad (4.13)$$

normalized as

$$1 = \int_{-\pi R}^{\pi R} dy (1 + a_I \delta_I) f_n^2. \quad (4.14)$$

The operator in (4.13) is  $\mathcal{O}_s = \partial_5^2 + \frac{c_I}{\Lambda} \partial_5 \delta_I \partial_5 + \frac{\kappa}{\Lambda^2} \partial_5^4$ . Using analytic renormalization it is possible to reduce this problem to a fermionic one. Indeed, after renormalization, the ‘‘supersymmetric’’ operator  $\tilde{\mathcal{O}}_s = -\mathcal{O}_f^\dagger \mathcal{O}_f$ , with  $\mathcal{O}_f = -(1 + \frac{c_I}{2\Lambda} \delta_I) \partial_5 - \frac{\kappa}{2\Lambda^2} \partial_5^3$ , is identical to  $\mathcal{O}_s$ , to second order. On the other hand, if  $f_{1n}$ ,  $f_{2n}$  and  $m_n$  are solutions of the fermionic eigensystem

$$\begin{aligned} \mathcal{O}_f^\dagger f_{2n} &= m_n(1 + a_I \delta_I) f_{1n}, \\ \mathcal{O}_f f_{1n} &= m_n f_{2n}, \end{aligned} \quad (4.15)$$

then  $f_n = f_{1n}$  and  $m_n$  are obviously solutions of  $\tilde{\mathcal{O}}_s f_n = -m_n^2(1 + a_I \delta_I) f_n$ , and hence of (4.13). Finally, by field redefinitions and analytic renormalization, we can eliminate the derivatives of delta functions and reduce (4.15) to (4.4) with  $a_I^L = a_I$ ,  $a_I^R = -c_I$ ,  $\kappa_1 = 0$  and  $\kappa_2 = -\kappa/2$ . The role of  $f_{1n}$  is played by  $f_n^L$ . Therefore, the results obtained above give the KK decomposition of an odd scalar. In particular, there is no zero mode. For an even scalar we can change  $\mathcal{O}_f \rightarrow \mathcal{O}_f^\dagger$  above, which leads to  $f_{1n} = f_n^R$  and parameters  $a_I^L = -c_I$ ,  $a_I^R = a_I$ ,  $\kappa_1 = 0$  and  $\kappa_2 = -\kappa/2$ .

We find that the KK masses are given by (4.7) with  $A = -\frac{a_0 + a_\pi}{2\pi}$  for an even scalar,  $A = \frac{c_0 + c_\pi}{2\pi}$  for an odd one, and  $B = -\frac{\kappa}{2}$  in both cases. Note that the terms with coefficients  $c_I$  give a non-trivial contribution for odd scalars, despite the fact that they do not contribute when treated non-perturbatively without renormalization [12, 24]. The wave functions also follow directly from the fermionic ones  $f_n^{L,R}$  in (4.8) and (4.9), using the same particular values for the fermionic parameters.

### 4.3 Gauge bosons

The case of gauge bosons is a special case of the scalar one when the gauge  $A_5 = 0$  is chosen. The main difference is that the most singular brane terms are forbidden by gauge invariance, but after analytic renormalization the free Lagrangians are equivalent. Indeed, after some field redefinitions the most general gauge kinetic Lagrangian to second order is  $\mathcal{L}_g = \mathcal{L}_g^{(0)} + \frac{1}{\Lambda} \mathcal{L}_g^{(1)} + \frac{1}{\Lambda^2} \mathcal{L}_g^{(2)} + \dots$ , with

$$\mathcal{L}_g^{(0)} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} F_{\mu 5} F^{\mu 5}, \quad (4.16)$$

$$\mathcal{L}_g^{(1)} = -\frac{1}{4} a_I \delta_I F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} c_I \delta_I F_{\mu 5} F^{\mu 5}, \quad (4.17)$$

$$\mathcal{L}_g^{(2)} = -\frac{1}{2} \kappa F_{\mu 5} \partial_5^2 F^{\mu 5}, \quad (4.18)$$

which, upon writing  $F_{MN} = \partial_{[M}A_{N]}$  and putting  $A_5 = 0$ , is identical to the real-scalar version of (4.12), with  $\phi \rightarrow A_\mu$ . Therefore, the KK reduction of  $A_\mu$  gives the same expressions.

## 5. Matching with fundamental theories

These results in the effective theory should agree with those obtained from a more fundamental theory in which the physics around the fixed points is non-singular. This is the case of weakly coupled string theory, where the extended nature of the strings softens the orbifold singularities. In perturbative string theory on an orbifold, some of the contributions of string loop corrections to a given correlation function are localized around the fixed points. These contributions are suppressed by powers of the string coupling, and the localization profile is controlled by the string length  $l_s$ , which is the only dimensionful parameter at hand (we are assuming a large compactification radius, so that finite-size effects are small). Therefore, even if we start with unresolved orbifolds with  $\epsilon_0 = 0$ , effectively this is smeared to  $\epsilon_0 \simeq l_s$ . These calculations have been performed explicitly for localized gauge-field tadpoles in heterotic string theory in [25] (see also [26]–[29]). Even if the corresponding result in field theory is non-singular, these calculations illustrate how an effective brane thickness is generated by the stringy dynamics near the fixed point, in agreement with our dimensional analysis (in fact, a dimensionless factor arising from normal-ordering constants in the world-sheet field theory turns out to be crucial as well). We refer to [30] for details about the analogies and differences in the string and field-theory calculations. It is plausible that a similar mechanism will also regulate quantities which in field theory contain thin-brane singularities. Other “fundamental theories” in which the physics around the fixed points is smooth are the field-theoretical orbifold resolutions of [31], which should be regarded as effective theories valid up to a cutoff  $\Lambda_0$  larger than the inverse size of the resolved fixed points,  $\epsilon_0^{-1}$ . In this case, there can be localized terms involving the curvature and the flux backgrounds.

To be more quantitative, we compare the renormalized effective theory with another ultraviolet completion: a deconstructed version [32, 33] of the orbifold  $S^1/Z_2$ . Deconstructed orbifolds were introduced in [33] (in the fundamental region) and will be studied in greater detail in [1] (starting from the parent theory space). They are renormalizable 4D theories in which the gauge group is formed by a product of identical simple groups, with “link” scalar fields charged under “neighbouring” pairs of group factors. Below the scale of the vacuum expectation value of the link scalars  $v$ , they are equivalent to 5D field theories whose fifth dimension is a latticized segment. The lattice spacing is  $s = (gv)^{-1}$ , with  $g$  a dimensionless coupling, and the radius  $R$  of the discrete  $S^1$  is given by  $\pi R = Ns$ , where  $N$  is the number of sites in the segment (the fundamental region). Scalar and fermion fields can also be added at each site to represent bulk scalars and fermions. We use Wilson fermions to avoid doubling, and fine-tune the parameters in such a way that chiral invariance is recovered in the continuum limit. The sites near the boundary of the interval behave differently from those deep inside the bulk, and localized terms are generated by quantum corrections, with coefficients independent of  $s$  and  $N$ . In this scenario, the fixed

points (boundaries) are described by Kronecker, rather than Dirac, deltas. The effect of the brane terms is thus regulated by the lattice spacing, which acts as a cutoff in position space. Therefore, we effectively have  $\epsilon_0 = \Lambda_0^{-1} \equiv s$ . In deconstruction, KK reduction amounts to a diagonalization of the mass matrix arising from the discrete kinetic term. The deconstructed orbifold theory can be described at energies below  $\Lambda = 1/s$  by a (classically) renormalized effective theory in a continuous orbifold with cutoff  $\Lambda$ . This means, in particular, that the KK masses and the discrete wave functions agree with the general ones we have found here. To see this, we must expand them to second order in a Taylor series about  $s = 0$ , keeping  $R$  fixed. Let us summarize the results.

For massless fermions, gauge bosons, massless odd scalars and generic massless even scalars, we find the following KK masses to second order [1]:

$$m_n = \frac{n}{R} \left[ 1 + \mathcal{A} \frac{s}{R} + \left( \mathcal{A}^2 - \frac{n^2 \pi^2}{24} \right) \left( \frac{s}{R} \right)^2 \right] + \dots, \quad (5.1)$$

where the value of  $\mathcal{A}$  depends on the kind of field and is a function of the coefficients of different operators near the fixed points in the deconstructed theory. For even gauge bosons and a fine-tuned class of even scalars — in which certain combinations of brane coefficients are put to zero — there is a flat zero mode. On the other hand, for generic even scalars the zero mode disappears and we find instead two tachyons, one localized at each brane. Their mass is proportional to the inverse spacing. For fermions there is a flat chiral zero mode; in some deconstructed orbifolds, there is in addition one zero mode localized at one of the branes, which has the same chirality as the bulk mode for “chiral” deconstructed orbifolds, and opposite chirality for “non-chiral” orbifolds. To next-to-leading order, the wave functions for massive KK modes of ( $Z_2$ -even) right-handed massless fermions, even gauge bosons and fine-tuned even massless scalars read

$$f_n^{(1)} = \left( N'_0 + N'_1 \frac{s}{R} \right) \cos \left( \frac{ny}{R} \right) - N'_0 \frac{n}{R\Lambda} \left( \mathcal{C}_1 + \mathcal{A} \frac{y}{R} \right) \sin \left( \frac{ny}{R} \right), \quad (5.2)$$

where  $y = is$  and  $i$  labels the sites. On the other hand, the wave functions of the corresponding (odd) left-handed fermions and of odd gauge bosons, odd massless scalars and generic even massless scalars are

$$f_n^{(2)} = -N'_0 \frac{n}{R\Lambda} \left( \mathcal{C}_2 + \mathcal{A} \frac{y}{R} \right) \cos \left( \frac{ny}{R} \right) - \left( N'_0 + N'_1 \frac{s}{R} \right) \sin \left( \frac{ny}{R} \right). \quad (5.3)$$

Here,  $N'_{0,1}$  are normalization constants and  $\mathcal{A}$  is the same expression (for each case) as appears in the masses.

All these results can be reproduced by the renormalized effective theories in the continuum. Indeed, eq. (5.1) neatly matches the generic expression (4.7) and the first-order deconstructed wave functions above have the same form as the ones in (4.8) and (4.9). Adjusting the parameters of the effective Lagrangians, we find exact agreement for KK masses and the corresponding wave functions in each case. The exact values in terms of the parameters of the different deconstructed models will be given in [1]. Finally, the localized zero modes and tachyons can be described directly in the effective theory by massless and tachyonic fields, respectively, living on the branes.

Interestingly enough, it turns out that the deconstructed generic even scalars are described by odd scalars (plus the brane tachyons) in the effective theory. This is due to the presence of discrete brane operators, which look like irrelevant from naïve power counting, but turn out to be relevant and change drastically the continuum limit. Hence, these theories belong to the same universality class as that of deconstructed odd scalars, except for the brane instability. When these operators are put to zero, the theory stays in the universality class one would have naïvely guessed. An alternative effective description is to use even scalars and add tachyonic boundary masses of the order of  $\Lambda$ . These are Dirac delta well potentials, which localize one mode — the tachyon — at each brane. The remaining KK modes are expelled from the branes by orthogonality.<sup>6</sup> Even though changing the parity of the field is simpler, the description of this effect by explicit relevant operators has several advantages. First, their behaviour under important symmetries can be studied. Second, their coefficients are dimensionful, and thus naturally of the order of the cutoff. This shows that effective Dirichlet boundary conditions for even scalars are natural. Third, the coefficients can be fine-tuned to be much smaller than the cutoff, in order to reproduce the fine-tuned scenario with effective Neumann boundary conditions. And fourth, their running in the effective theory can be studied with standard methods.

It might be thought that the non-decoupling effects should be related to the instabilities. However, there are stable examples in which this change of boundary conditions in the infrared is observed. For instance, in the non-chiral class of deconstructed fermions, if we do not fine-tune the mass and Wilson term near the boundaries (as has been assumed so far), both zero modes combine to form a massive Dirac mode [1]. The full set of KK masses is then given by

$$m_n = \frac{n + 1/2}{R} \left[ 1 + \mathcal{B} \frac{s}{R} + \left( \mathcal{B}^2 - \frac{(n + 1/2)^2 \pi^2}{24} \right) \left( \frac{s}{R} \right)^2 \right] + \dots, \quad (5.4)$$

while the wave functions of the right-handed fermion are

$$f_n^{(3)} = \left( N'_0 + N'_1 \frac{s}{R} \right) \cos \left( \frac{(n + 1/2)y}{R} \right) - N'_0 \frac{n + 1/2}{R\Lambda} \left( \mathcal{C}_3 + \mathcal{B} \frac{y}{R} \right) \sin \left( \frac{(n + 1/2)y}{R} \right), \quad (5.5)$$

to first order. The same form of the masses and wave functions, plus one localized tachyon, is obtained for deconstructed even scalars with the mentioned operators adjusted to zero only near the boundary  $y = 0$ . This behaviour can be precisely matched to an effective theory in which different boundary conditions are used at the two boundaries for each chiral component (Neumann-Dirichlet at lowest order). In the orbifold formalism in the “parent” space, this can be achieved by allowing for a twist such that the field has antiperiodic boundary conditions, or equivalently, using an orbifold  $S^1/(Z_2 \times Z'_2)$ . As in the scalar case, an alternative description preserving the original parity of the fields is possible, if relevant operators are included. Specifically, the operator doing the job is a mass mixing between

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<sup>6</sup>A big non-tachyonic boundary mass also gives rise to Dirichlet boundary conditions but does not localize any mode.

the right-handed bulk fermion and a left-handed localized mode at one of the branes (which was present in the purely massless case). Its coefficient has mass dimension  $1/2$ , and is naturally of order  $\Lambda^{1/2}$ . This operator, which breaks the chiral invariance of the bulk fermion, mimics faithfully the physical mechanism involved in the change of the continuum boundary conditions of the deconstructed theory. More details will be given in [1].

## 6. Odd-odd operators

We have seen that operators containing a delta function times a product of odd functions (odd-odd brane terms) do not have any effect to second order in the effective theory. The same holds in deconstruction, but these terms start contributing at third order in  $s$ . If we want to reproduce this effect with odd-odd terms in the effective theory we need to define the convolution of delta functions and discontinuous functions. We can perform a formal calculation by treating the value of  $f_n^R$  at  $y = 0, \pi R$  as independent from the corresponding limits and with the prescription that for discontinuous functions  $g$ ,

$$\int dy \delta(y - y_0) g(y) = \frac{c}{2} \left( \lim_{y \rightarrow y_0^+} g(y) + \lim_{y \rightarrow y_0^-} g(y) \right) + (1 - c)g(y_0), \quad (6.1)$$

with  $0 < c < 1$ . Then, we find boundary conditions from the even equation in the usual way, but using (6.1). On the other hand, we multiply the odd equation by a periodic sign function  $\sigma$  (with  $\sigma(0, \pi R) = 0$ ) and integrate around  $y = 0, \pi R$ , using again (6.1). This gives a second pair of boundary conditions relating  $f_n^R(0, \pi R)$  with the limits of  $f_n^R$ , which allows to write boundary conditions in terms of the limits of  $f_n^R$  and  $f_n^L$  only. In this way, we find that the third-order contribution to the fermion KK masses of the (odd-odd) terms proportional to  $a_{0,\pi}^L$  is

$$-c \frac{n^3}{8\pi R^4 \Lambda^3} [(a_0^R)^2 a_0^L + (a_\pi^R)^2 a_\pi^L]. \quad (6.2)$$

Note that the numerical coefficient is regularization dependent. This is not a problem, as this dependence can be absorbed into the renormalized couplings  $a_I^L$ . In order to justify this formal computation, we look now at the calculation with an explicit regulator. If we introduce a dimensionful regulator, we must be careful that it do not mix with the expansion in  $\Lambda$  and reintroduce the thin-brane singularities. We have found explicitly that the “point-splitting” regulator introduced in [34] does not have this problem. This regularization consists in shifting the support of the delta functions a distance  $\epsilon$  away from the fixed points, performing the calculations and taking  $\epsilon \rightarrow 0$  at the end. To deal with the discontinuous functions at  $\epsilon$  we use the prescription (6.1) with  $c = 1$ , which in particular follows from the analytic regularization in the appendix.<sup>7</sup> Then we find

$$-\frac{n^3}{32\pi R^4 \Lambda^3} [(a_0^R)^2 a_0^L + (a_\pi^R)^2 a_\pi^L], \quad (6.3)$$

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<sup>7</sup>In principle, we could just use the regularization in the appendix, without point splitting, but then we would have to solve the equations of motion in a background involving polylogarithms, which is not an easy task. On the other hand, setting  $c = 1$  in the formal calculation above gives inconsistent equations, but it is possible to take the limit  $c \rightarrow 1$  at the end.

which coincides with (6.2) for  $c = 1/4$ . In any case, the contribution of  $a_I^L$  in (6.2) vanishes if  $a_I^R$  (with the same  $I$ ) does. The reason for this is that the odd-odd terms contribute only when the fields are discontinuous at the branes, and this discontinuity is induced at lower orders by  $a_I^R$ . The dependence on  $n$  and  $R$  matches the one of the corresponding contribution obtained in deconstruction. However, in this case the even-even brane terms, which give rise to  $a_I^R$  in the continuum, do not need to be turned on to have a non-vanishing contribution of the third-order odd-odd term. The effective theory can also reproduce such correction with  $a_I^R = 0$  by means of the third-order operator  $\delta_I \bar{\psi}_R \partial_y^3 \bar{\psi}_L$ , which gives a contribution with the same  $R$  and  $n$  dependence as (6.2). This shows that, at least as far as the KK masses are concerned, the operators with coefficient  $a_I^L$  are redundant to third order.

One might speculate that this is a general property of the free effective theory, i.e. that the effect of any odd-odd term, to all orders, can be absorbed into higher-order even-even terms. We have not found a field redefinition showing this, and in fact field redefinitions preserving the orbifold parity will not mix odd-odd with even-even operators. However the possibility that odd-odd terms be redundant agrees with the idea that the free brane operators simply determine the boundary conditions outside the core of the brane [11], and an arbitrary boundary condition can be imposed by adjusting the value of the even-even operators. From this argument, however, it does not follow that the correct dependence on the KK number at each order will be reproduced. On the other hand, in the interacting effective theory the odd-odd operators run in general with the renormalization group scale, and this running would have to be incorporated into the even-even operators. This would require an explicit relation between even-even and odd-odd operators. These issues deserve further study.

## 7. Conclusions

We have argued, and showed explicitly in particular models, that effective theories in extra-dimensional spaces with infinitely thin defects are a good description of more fundamental theories in which the defects can have some structure. A renormalization procedure is necessary to take care of the divergences which appear in the thin-brane limit and we have proposed a simple renormalization prescription for plane branes of codimension 1, analytic renormalization, which defines these divergences as vanishing. We have shown that, in this scheme, even-even orthogonal brane kinetic terms can be completely eliminated by a field redefinition. Odd-odd parallel and orthogonal terms can also be disregarded to second order and maybe higher. As a matter of fact, only even-even parallel brane kinetic terms (besides mass brane terms) are customarily taken into account in phenomenological fits [35]–[41] and model building [42]–[46]. Our results imply that, in the framework of a classically renormalized effective theory, this is consistent and does not entail a loss of generality. Moreover, it agrees with completions such as deconstructed orbifolds. The less common works including orthogonal terms use fat branes [47] or treat the orthogonal brane terms perturbatively to first order [9], which is non-singular. The first possibility reduces to the bare effective theory at scales lower than the physical width of the branes.



Regarding the second one, we have shown that the first-order results in [9] are not spoiled by divergent higher-order contributions if perturbative renormalization is implemented.

It should be observed that all our results are perturbative in the derivative expansion and assume that dimensionless couplings are of order 1. However, parallel brane kinetic terms with coefficients larger than  $R$  are often invoked. This puts the theory in a non-perturbative regime. Even though the effect of the large parallel kinetic terms (of first order, formally) can be resummed to all orders, the result may be changed by contributions of higher-order operators, which can be of the same size in principle. It can still be assumed that all higher-order terms vanish or have small coefficients, although this choice is not protected by any symmetry. From this point of view, the calculations with small brane terms, as those in universal extra dimensions [9, 48], are more robust.

On the other hand, we have seen in explicit examples that the renormalized effective formalism can also describe, perturbatively, scenarios in which the infrared behaviour is abruptly changed when certain operators in the fundamental theory are turned on. This effect is reproduced in the effective theory by relevant operators, with coefficients which are naturally of the size of the cutoff — but can be smaller if the symmetry is enhanced when they vanish. We remark that similar effects would be produced by certain irrelevant operators in the effective theory at the regularized level, if the divergences were not subtracted. From this point of view, classical renormalization in a mass-independent scheme can be understood as a reorganization of the effective theory, such that the impact of each operator is controlled by the size of its coefficient. This allows us to work at a fixed order consistently, since the contribution of higher-order operators to a given observable is guaranteed to be smaller (as long as their dimensionless coefficients are of order 1). On the other hand, the relevant operators can be alternatively represented by different orbifold field transformations (or boundary conditions), at least at the classical level.

In this paper we have focussed on the free local sector of effective theories and their deconstructed completions, although we have in mind the possibility that some of the operators may be partially or fully induced by quantum corrections when interactions are included. Before concluding, let us make a few comments on the interactive theory. As long as a complete set operators is included in the effective theory, we should be able to reproduce the interactions of completions such as the deconstructed models. The tree graphs will have in general new thin-brane singularities, which can be subtracted with analytic renormalization. Odd-odd terms appear too, and can be treated as discussed above.<sup>8</sup> In [6, 7] complete quantum computations in renormalizable field theories of dimension 4 with boundaries have been performed, and the corresponding renormalization group equations have been studied. The techniques in these references can be applied to theories of dimension higher than 4, although in this case the singularities will be more severe and more counterterms will be required because of non-renormalizability. There are many examples of loop calculations in extra dimensions with branes in the literature. See for instance [8, 9, 21, 48–52]. At the quantum level, one must take care of

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<sup>8</sup>Bulk higher-derivative operators giving rise to singularities and ill-defined products can be redefined away using the classical equations of motion, as shown in [11].

the usual “quantum” UV divergences at coincident points, which can be divided into bulk and brane-localized divergences, and also of the thin-brane “classical” divergences we have been discussing so far. In our perturbative effective framework, the quantum divergences should also be treated in a mass-independent renormalization scheme, such as dimensional regularization with minimal subtraction, so that the renormalization scale  $\mu$  only appears inside logarithms, and does not interfere with the counting of powers of  $\Lambda$ . In this case, the localized quantum divergences are proportional to exact delta functions and can be cancelled by brane counterterms in the effective theory with vanishing brane width [6–8] (for examples showing the smooth profile of divergences with a hard cutoff, see [49, 53]). The thin-brane divergences, on the other hand, which appear typically in one-particle reducible (sub)diagrams, can be subtracted using our prescription. In general, the perturbative renormalization of both classical and quantum divergences in extra-dimensional theories with branes can be organized along the lines of appendix B in [6]. Finally, if a matching with a fundamental theory is performed, this should be carried out, as usual, at a renormalization scale  $\mu = \Lambda$ . Then, the renormalization group equations of the effective theory can be used to calculate processes at lower energies.

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## A. Explicit regularization

Here we give one example of analytic regularization which naturally leads to our renormalization prescription,  $\prod_{j=1}^m \delta^{(n_j)}(0) = 0$  for  $m \geq 1$  (with the superscripts  $n_j \geq 0$  indicating the order of the derivatives), without any explicit subtraction. First, we observe that, in the noncompact case, any regularization by analytic continuation of an adimensional parameter must give a vanishing result for  $\delta(0)$  and similar integrals, just for dimensional reasons.<sup>9</sup> However, in the compact case we can use the compactification radius to build dimensionless quantities, and different results are possible. In general, divergences will appear in the limit in which the regulator is removed, and a renormalization prescription (such as minimal subtraction of poles, for instance) must be supplied to subtract them. In the regularization we propose below, these integrals not only are finite, but they actually vanish. This simplifies the calculations, for one does not have to worry about finite parts arising from cancellations of poles and zeros.

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<sup>9</sup>In the case of dimensional regularization, there is no value of the complex dimension for which integrals like  $\int dk k^{-n}$  converge, and a vanishing value is assigned by definition. Nevertheless, it is possible to generalize the regularization so that a finite region exists for which the integral converges [54]. Then, analytic continuation gives indeed a vanishing value.

Let us parametrize the extra-dimensional coordinate, in the compact covering space, by the dimensionless coordinate  $x = y/R \in ] - \pi, \pi]$ . Consider the following representation of the delta function,

$$\delta_t(x) = \frac{1}{2\pi} [1 + \text{Li}_t(e^{ix}) + \text{Li}_t(e^{-ix})] , \tag{A.1}$$

where  $\text{Li}$  is the polylogarithm [55], which is defined for complex numbers  $t$  and  $z$  with  $|z| < 1$  by

$$\text{Li}_t(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^t} . \tag{A.2}$$

Its definition for  $|z| \geq 1$  follows uniquely via analytic continuation. For a generic fixed  $t$ , the polylogarithm is multi-valued. We choose the principal branch with  $\text{Li}_t(z)$  real for  $z$  real,  $0 \leq z \leq 1$ , and continuous except for a cut along the real axis from  $z = 1$  to  $z = \infty$ . The discontinuity across the cut is

$$\text{Li}_t(z + i0^+) - \text{Li}_t(z + i0^-) = 2i \frac{\pi(\log z)^{t-1}}{\Gamma(t)} , \tag{A.3}$$

for real  $z \geq 1$ , where the logarithm is evaluated in its principal branch with  $-\pi < \arg(z) \leq \pi$ .

For fixed  $t$  with  $\text{Re}(t) > 1$ , the function  $\delta_t(x)$  is continuous in  $x \in ] - \pi, \pi]$ . When  $0 < \text{Re}(t) \leq 1$ , there is an integrable singularity at  $x = 0$  (logarithmic for  $t = 1$ ). The indefinite integral of the regularized delta can be computed for any  $t$ :

$$\int dx \delta_t(x) = \frac{1}{2\pi} [x + i\text{Li}_{t+1}(e^{ix}) - i\text{Li}_{t+1}(e^{-ix})] . \tag{A.4}$$

Hence,  $\delta_t$  satisfies

$$\int_{-\pi}^{\pi} \delta_t(y) = 1 . \tag{A.5}$$

Of course, the same result can be obtained directly from the corresponding contour integral. On the other hand,  $\delta_0(x) = 0$  if  $x \neq 0$ , so  $\delta_t$  goes to a Dirac delta function as  $t \rightarrow 0$ . Then, we define the integral of a test function times a delta by the analytic continuation to  $t = 0$  of the integral evaluated for  $\text{Re}(t) > 0$ . For well-behaved test functions, this just gives the value of the function evaluated at  $x = 0$ . For a test function discontinuous at  $x = 0$  it also gives a definite result:

$$\int_{-a}^a \varphi(x) \delta(x) = \frac{1}{2} \left( \lim_{x \rightarrow 0^-} \varphi(x) + \lim_{x \rightarrow 0^+} \varphi(x) \right) , \tag{A.6}$$

with  $0 < a \leq \pi$ . This provides a simple prescription to handle discontinuous functions, which arise often in the presence of brane terms. The  $n$ -th derivative of the regularized delta is

$$\delta_t^{(n)}(x) = \frac{i^n}{2\pi} (\text{Li}_{t-n}(e^{ix}) + (-1)^n \text{Li}_{t-n}(e^{-ix})) . \tag{A.7}$$

It is continuous in  $] - \pi, \pi]$  when  $\text{Re}(t) > 1 + n$ . To convolute  $\delta^{(n)}$  with a test function, it is sufficient to take  $\text{Re}(t) > n$ . Alternatively, if the test function can be differentiated  $n$

times, we can simply integrate by parts, as in the formal definition of the  $n$ -th derivative of the delta function.

When products of delta functions occur, we extend our definition to

$$\int_a^b dx \left[ \prod_{j=1}^m \delta^{(n_j)}(x) \right] \varphi(x) = \lim_{t \rightarrow 0} \int_a^b dx \left[ \prod_{j=1}^m \delta_t^{(n_j)}(x) \right] \varphi(x), \quad (\text{A.8})$$

where  $\lim$  indicates the analytic continuation from sufficiently large  $\text{Re}(t)$ .

For  $\text{Re}(t) > 1$ , we can use (A.2) to write (A.1) as

$$\delta_t(x) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} e^{-ikx} \xi_t(k), \quad (\text{A.9})$$

with  $\xi_t(k) = |k|^{-t}$  if  $k \neq 0$  and  $\xi_t(0) = 1$ . This is just a zeta-function regularization [56–58] of the Fourier series of the delta function. In this representation, the regularization allows to interchange the sum and integral signs, so that

$$\int_{-\pi}^{\pi} dx \delta_t(x) \varphi(x) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \xi_t(k) \hat{\varphi}(k), \quad (\text{A.10})$$

where  $\hat{\varphi}$  is the Fourier transform of  $\varphi$ . If the sum in the r.h.s. of (A.10) is convergent for  $t = 0$ , we can remove the regularization before performing the sum, and again we see that we recover the Dirac delta. Doing the exact sum corresponding to the integral of the regularized  $\delta^2$ , we obtain

$$\begin{aligned} \int_{-\pi}^{\pi} dx \delta_t(x)^2 &= \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \xi_t(k)^2 \\ &= \frac{1}{2\pi} (1 + 2\zeta(2t)), \end{aligned} \quad (\text{A.11})$$

with  $\zeta$  the Riemann zeta function. Continuing to  $t = 0$  and using  $\zeta(0) = -1/2$ , we find  $\int_{-\pi}^{\pi} \delta(y)^2 = 0$ . In fact, this result can be found from the position space representation in a more formal way:

$$\begin{aligned} \int_{-a}^a dx \delta_t(x)^2 &\rightarrow \int_{-a}^a dx \delta_t(x) \delta(x) \\ &= \delta_t(0) \\ &= \frac{1}{2\pi} (1 + 2\zeta(t)), \end{aligned} \quad (\text{A.12})$$

which again gives 0 at  $t = 0$ . In the first line we have taken  $t = 0$  in one of the deltas and kept  $\text{Re}(t) > 1$  in the other. This trick, which is equivalent to exchanging the limit and the sum in the Fourier representation, works because  $\delta_t(x)$  is well behaved when  $\text{Re}(t) > 1$ . In the same way, taking  $t = 0$  in one of the deltas before performing the integration, we find

$$\int_{-a}^a \delta(x)^m \varphi(x) = 0, \quad m \geq 2. \quad (\text{A.13})$$

Moreover, for  $\text{Re}(t) > n + 1$ ,

$$\delta_t^{(n)}(0) = \begin{cases} \frac{\zeta(t-n)}{\pi}, & n \text{ even,} \\ 0, & n \text{ odd.} \end{cases} \quad (\text{A.14})$$

Since  $\zeta(t)$  vanishes at negative even integers, the analytic continuation to  $t = 0$  gives  $\delta^{(n)}(0) = 0$ . More generally, for any  $a$  and  $b$ ,

$$\int_a^b \left[ \prod_{j=1}^m \delta^{(n_j)}(x) \right] \varphi(x) = 0, \quad m \geq 2. \quad (\text{A.15})$$

## B. Vector formulation of 5D Green-Schwarz mechanism

As a further test of our renormalization scheme, and to illustrate how it can be used to simplify calculations in which the thin-brane singularities cancel, we compute the spectrum of a theory describing a five-dimensional Green-Schwarz (GS) mechanism in the vector formulation [22]. The five-dimensional GS mechanism can be described in two dual formulations, using tensor and vector fields, respectively. The former is explicitly free of singularities, whereas the latter has  $\delta(0)$  terms at intermediate steps of the calculation. They must cancel out in the final results, in such a way that the physical observables agree in both descriptions. One can then use this duality to test the regularization of the singular terms, as has been done in [22]. In this appendix, we show that our prescription of renormalizing the singularities to zero does give the correct result for the spectrum, without the need of an explicit regularization.

The relevant Lagrangian, at the quadratic level, reads

$$\mathcal{L} = -\frac{1}{4} F_{MN} F^{MN} - \frac{1}{4} \mathcal{F}_{MN} \mathcal{F}^{MN} + \sum_I \delta_I \xi_I (A^\mu \mathcal{F}_{\mu 5} - \frac{1}{2} \xi_I \delta_I A_\mu A^\mu), \quad (\text{B.1})$$

where  $A_\mu$  and  $\mathcal{A}_5$  ( $A_5$  and  $\mathcal{A}_\mu$ ) are even (odd) fields under the  $Z_2$  parity. Note that the normalization of the delta functions is different from the one in [22], as we work in the circle picture rather than the interval one. Following our prescription, we set to zero the last term in the parentheses, which contains  $\delta(0)$ . If other  $\delta^2$  terms appear during the calculation they will also be dropped in a consistent way. We now take the gauge  $\partial^\mu A_\mu + \partial_5 A_5 = 0$  and  $\partial^\mu \mathcal{A}_\mu + \partial_5 \mathcal{A}_5 = 0$  to decouple the  $\mu$  from the 5 components of the gauge fields and introduce the following KK expansion

$$A_\mu(x, y) = \sum_n f_n(y) A_\mu^{(n)}(x), \quad (\text{B.2})$$

$$\mathcal{A}_\mu(x, y) = \sum_n g_n(y) \mathcal{A}_\mu^{(n)}(x). \quad (\text{B.3})$$

The resulting equations of motion for the different modes read

$$(\partial_5^2 + m_n^2) f_n = \delta_I \xi_I \partial_5 g_n, \quad (\text{B.4})$$

$$(\partial_5^2 + m_n^2) g_n = -\xi_I \partial_5 (\delta_I f_n), \quad (\text{B.5})$$

where we have denoted by  $m_n$  the eigenvalue of the four-dimensional momentum. The solution to the singular part of eq. (B.5) is

$$\partial_5 g_n = -\xi_I \delta_I f_n, \tag{B.6}$$

which can be integrated around the orbifold fixed points to give the boundary conditions

$$g_n(0^+) = -\frac{\xi_0}{2} f_n(0), \tag{B.7}$$

$$g_n(\pi R^-) = \frac{\xi_\pi}{2} f_n(\pi R). \tag{B.8}$$

Now the singular part of eq. (B.4) reads

$$\partial_5^2 f_n = \delta_I \xi_I \partial_5 g_n = \delta_I \xi_I (-\xi_I \delta_I f_n + \partial_5 \hat{g}_n) \rightarrow \delta_I \xi_I \partial_5 \hat{g}_n, \tag{B.9}$$

where in the second equality we have used eq. (B.6) and have defined  $\partial_5 \hat{g}_n \equiv \partial_5 g_n(\pi I^+)$  as its regular part, and in the last equality we have used our prescription to drop the term proportional to  $\delta(0)$ . Again, this equation can be integrated around the orbifold fixed points to give the second set of boundary conditions,

$$\partial_5 f_n(0^+) = \frac{\xi_0}{2} \partial_5 g_n(0^+), \tag{B.10}$$

$$\partial_5 f_n(\pi R^-) = -\frac{\xi_\pi}{2} \partial_5 g_n(\pi R^-). \tag{B.11}$$

The full solution is then given by the solution of the bulk equations,

$$f_n = A_n \cos(m_n y) + B_n \sin(m_n y), \tag{B.12}$$

$$g_n = C_n \cos(m_n y) + D_n \sin(m_n y), \tag{B.13}$$

subject to the four boundary conditions, eqs. (B.7), (B.8), (B.10), (B.11), which imply

$$\begin{aligned} 0 &= \frac{\xi_0}{2} A_n + C_n, \\ 0 &= B_n - \frac{\xi_0}{2} D_n, \end{aligned} \tag{B.14}$$

$$0 = \frac{\xi_\pi}{2} \cos(m_n \pi R) A_n + \frac{\xi_\pi}{2} \sin(m_n \pi R) B_n - \cos(m_n \pi R) C_n - \sin(m_n \pi R) D_n,$$

$$0 = \sin(m_n \pi R) A_n - \cos(m_n \pi R) B_n + \frac{\xi_\pi}{2} \sin(m_n \pi R) C_n - \frac{\xi_\pi}{2} \cos(m_n \pi R) D_n.$$

Non-trivial solutions of this homogeneous system of equations satisfy the following eigenvalue equation

$$\tan^2(m_n \pi R) = \frac{1}{4} \left( \frac{\xi_0 + \xi_\pi}{1 - \xi_0 \xi_\pi / 4} \right)^2, \tag{B.15}$$

which fully agrees with the result on reference [22]. This should not come to a surprise since, had we kept the explicit  $\delta(0)$  terms, eq. (B.4) would have been replaced by

$$(\partial_5^2 + m_n^2) f_n = \delta_I \xi_I (\partial_5 g_n + \xi_I \delta_I f_n), \tag{B.16}$$

and, by virtue of eq. (B.6), the explicit singular terms proportional to  $\delta(0)$  would have exactly cancelled, as they should, and we would have ended with the same final (non-singular) result. Just as an extra cross-check, we have solved the same eigenvalue problem with a box regularization of the delta function and again the same result (explicit cancellation of the  $\delta^2$  terms and correct eigenvalue equation) was obtained after taking the width of the regularized delta to zero.

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